AMYGDALA $z \mapsto z^2+c$

A Newsletter of fractals & $\mathcal{M}$ -- the Mandelbrot Set
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Issue #9
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SLIDES

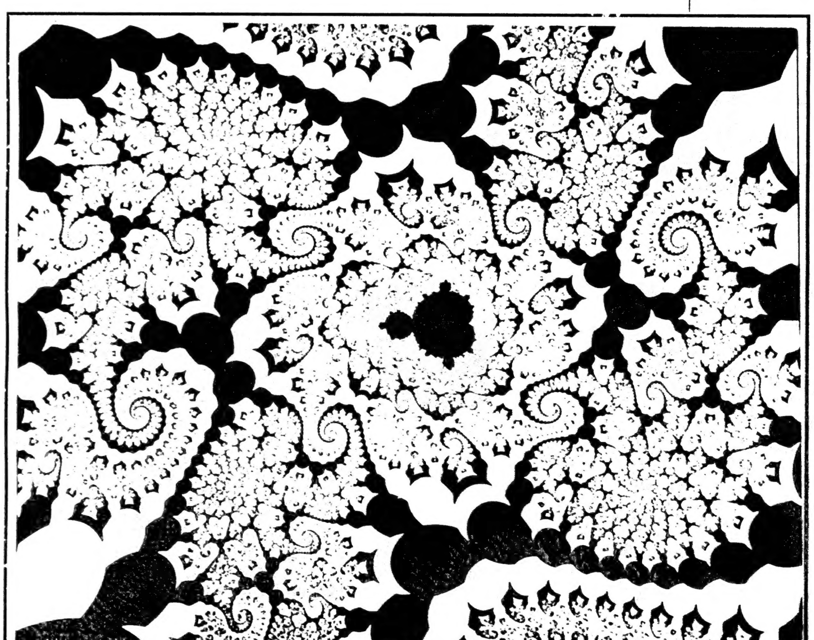
The color slide supplement continues with four more slides sent to subscribers with this issue.

#172 (John Dewey Jones): Looks like TBF Map 44, but with different colors, thus it would seem to be a detail of one of the infinity of seahorse tails inhabiting the Mandelbrot plenum.

#348 (Ken Philip): with tokens of office.
center = -1.711175+0.000384i, magnification = 2060, aspect ratio = 1.757, critical radius = 2, dwell limit = 64.

#349 (Ken Philip): 8-legged ant.
center = -1.7116300+0.0004400i, magnification=25000, critical radius = 2, aspect ratio = 1.000, dwell limit = 64.

#600: The color version of Charley Fitch's puzzle picture (see **HOW DID HE DO THAT?**, next). The coordinates are center = 0.776785376+ 0.13576353i, magnification = 3,690.



HOW DID HE DO THAT?

Charles M. Fitch

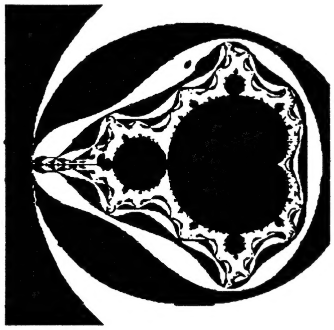
It is well known that by changing the boundary test for infinity from a circle of radius 2 to some other shape that contains the circle of radius 2, that one can change the shape of the contour lines in the potential field of M. I would like to challenge the readers to try this, and I will offer a silk-screened M T-shirt to the first one who discovers the boundary shape I used to create the wonderful overlapping petal shaped contour lines you see in the picture to the left [and in slide #600 included with the color subscription distribution with this issue]. Send in your solutions to this challenge to Amygdala.

The slide was computed at 2560 x 2048 x 16 bit resolution and anti-aliased to 1280 x 1024 x 24 bits for recording on a Matrix 3000 film recorder.

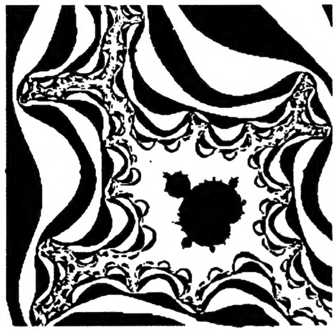
TIMING STUDY

In Amy #4 I promised to "publish performance

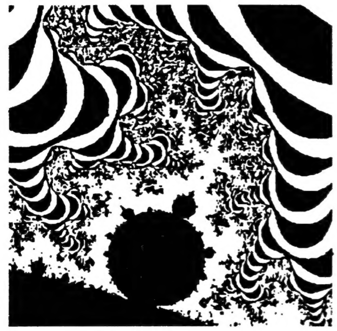
JDJ 27



Map 26

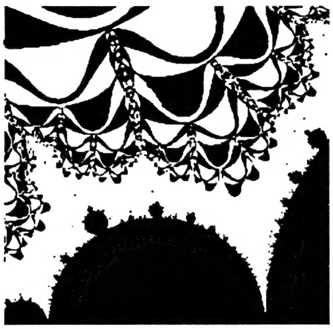


Map 27



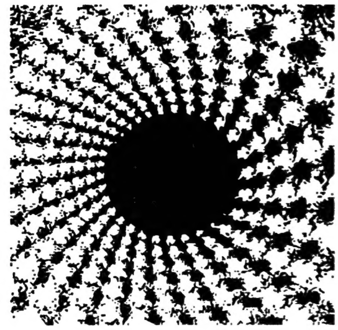
Map 29

Map 30



Map 33

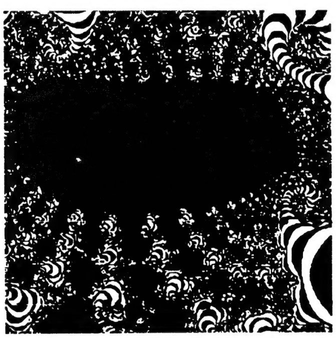
Map 36



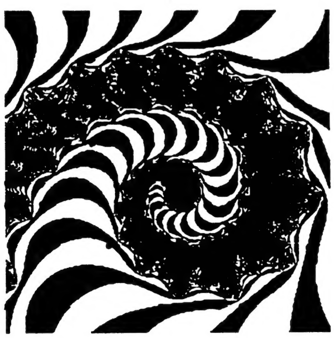
Map 38

figures for EMD on a number of Mandelbrot views in the next issue, to serve as a basis for a realistic appraisal of its performance." In fact I didn't publish them in #5, but here they are now. The comparison is between TVScan and DivCon, both of which use my fast iteration function written in Assembler, but differ in that TVScan computes the dwell of all pixels, while DivCon uses the modified Mariani algorithm (Amy #4, page 4) to evade such computation where possible.

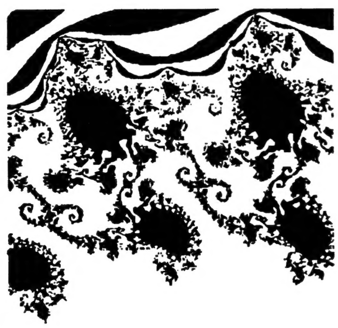
As noted in Amy #6, page 5, there is a bug in the fast iteration code; It results in the loss of a bit or so of accuracy, but I don't expect it to affect the appearance of the pictures or



Map 40

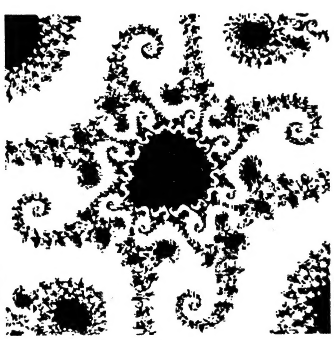


Map 42



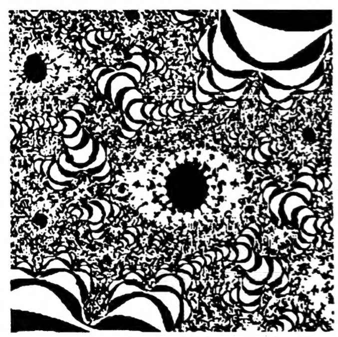
Map 44

Map 45



Map 48

Map 49



Map 58

MAP	CENTER	MAGNIF	LIMIT	TVS SEC	DivCon SEC	% INCR	NOTES
JDJ 27	-1.9963800	41,667	100	205	145	41%	
			1000	275	190	45%	
Map 26	-0.7500000	0.667	100	213	95	124%	
			1000	1,598	493	224%	T = 8
Map 27	-0.1643700+1.0409350i	38.3	100	190	156	22%	
			1000	500	285	75%	T = 18
Map 29	-0.9166650+0.2666650i	30	100	418	289	45%	
			1000	1,694	739	129%	T = 50
Map 30	-0.5606000+0.6032250i	9	100	321	196	64%	
			1000	1,709	617	177%	T = 24
Map 33	-1.7725000+0.0065000i	154	100	487	246	98%	
			1000	2,800	775	261%	
Map 36	-0.7459200+0.1102350i	195	100	669	354	89%	
			1000	2,442	1,798	36%	
Map 38	-0.7469100+0.1072500i	1,852	1000	3,984	4,018	-1%	
Map 40	-0.7464595+0.1076260i	19,231	1000	4,518	4,508	0%	
Map 42	-0.7451950+0.1126750i	1,399	100	682	466	46%	
			1000	969	859	13%	
Map 44	-0.7452960+0.1130585i	5,634	100	780	415	88%	
			1000	1,339	1,281	5%	
Map 45	-0.7454265+0.1130090i	33,333	100	904	316	186%	violin "S"
			1000	1,428	1,327	8%	
Map 48	-0.7454286+0.1130088i	196,078	1000	2,359	2,170	9%	filigree M
Map 49	-0.7454295+0.1130081i	1,000,000	100	2,252	757	197%	snout of #48
			2000	5,072	3,629	40%	
Map 58	-1.2534425+0.0466885i	2,291	150	707	686	3%	Medusae
Map 58	-1.2534425+0.0466885i	2,291	1000	749	745	1%	

the timing very much.

In addition to John Dewey Jones's #37 (the slide distributed with Amygdala #2), I have run EMD on almost all the maps in The Beauty of Fractals, pages 78-92, which are "c-Plane Pictures: Mandelbrot Set and Close-Ups", whose coordinate data are given on page 193, namely Maps 26, 27 (52,53,54), 29, 30, 33(51), 36, 38, 40, 42(100,101), 44 (46,99), 45(47), 48(50), 49, and 58(59,60,61). I have attempted to foil my natural tendency to favor DivCon by not selecting which maps to include. I have, of course, included only one representative of maps which are simply color variants of each other. The data given on page 193 have been converted to center and magnification. The percent speedup figures are just that: If TVScan takes 100 seconds and DivCon takes 50, that is a speedup of 100%.

All pictures were done with pixelation 251x251. The "coloring" scheme used to produce these pictures is rather crude. it utilizes two parameters, the dwell limit D, and a threshold dwell value T. Points with dwell d are colored as follows: if $d = D$, or $d \leq T$ and $D-d$ is even, color the point black; otherwise white.

The first thing that struck me about the figures is the wide variation in the effectiveness of DivCon, with speedup varying from 261% (for Map 33, dwell limit 1000) down to -1% (for Map 38). DivCon is effective when there is are relatively broad areas of constant large dwell, e.g. if there is a significant amount of $\mathcal{M}$ in the picture (e.g. Map 26). It is less effective if the areas of large dwell tend to be small. In Map 38 (d = 1000), for example, there are no large areas of constant dwell at all; DivCon never finds a rectangle larger than 1x1 with constant dwell on its perimeter. As a result, DivCon is ineffective, and its greater overhead results in a slight slowdown relative to TVScan.

Another remarkable[1] thing about Map 38 (and several others as well) is the uninteresting character of the B&W map compared to the color map. This may be due to the fact that the escape radius (see ESCAPE RADIUS in Amy #8) used in the B&W maps is 2, while that used in the color maps is probably 100. I rather suspect, though, that the interesting structure in the color map results from the artful grouping of nearby (but not identical) dwell values into neighboring color values, which my simpleminded B&W coloring scheme of alternating stripes cannot emulate.

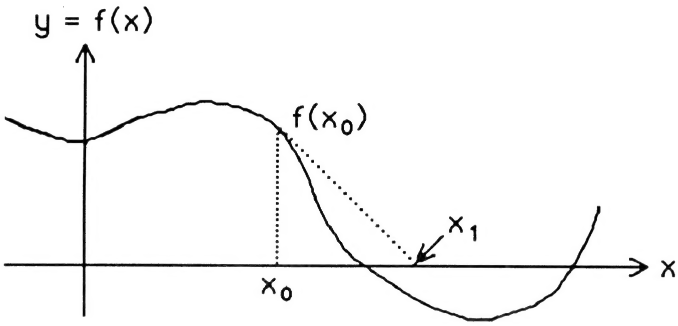
DivCon provides a modest improvement over TVScan for JDJ27, most of it because of the small $\mathcal{M}$ in the middle. The other areas, although broad, have such small dwell rela-

[1] Literally... and I am remarking it.

tive to the M that they don't take up much of the overall time, even for TVScan.

NEWTON'S METHOD

Newton's method is an algorithm for finding roots f(x) = 0 of an arbitrary smooth function, provided that you can find a sufficiently good approximation to a root to start with. The idea is to extrapolate down the tangent to the curve of the function at each approximation, to get a better one:



Here, given an approximation x_0 to a root of the function f, we have $f(x_0) \neq 0$, so we wish to improve our estimate of the root. The equation of the tangent line to the curve through the point $\langle x_0, f(x_0) \rangle$ is:

$$[y - f(x_0)]/(x-x_0) = f'(x_0) \qquad (1)$$

where $f'(x_0)$ is the derivative df/dx of the function f at the point x_0. Our new approximation x_1 will be the value of x for which y = 0: $0-f(x_0) = f'(x_0)(x_1-x_0)$, or

$$x_1 = x_0 - f(x_0)/f'(x_0) \qquad (2)$$

which is Newton's formula.
We can express this in terms of the iteration operator '$\mapsto$':

$$x \mapsto x - f(x)/f'(x) \qquad (3)$$

For example, to find the square root of a number **a** is to find a root of $f(x) = x^2-a$. Here the derivative f'(x) is 2x, and the iteration (3) becomes $x \mapsto x - (x^2-a)/2x$, or:

$$x \mapsto (x + a/x)/2 \qquad (4)$$

This is, by the way, a splendid way to compute the square root. Given a fairly good initial approximation, each iteration doubles the number of digits of accuracy!

Newton's method (3) is valid when extended to functions of a complex variable z:

$$z \mapsto z - f(z)/f'(z) \qquad (5)$$

and this is the basis of various iterations that some readers have found "bizarre looking". Chapter 6 of The Beauty of Fractals has details of using this iteration not for root-finding ("Newton's method in the small") but for exploring the "basins of attraction" of the roots of equations under iteration ("Newton's method in the large").

The continuing tutorial in *Amygdala* should give you a good idea of how to separate real and imaginary elements in elaborate formulae so that they may be computed, but I'll review that here.

Given z = x+iy and w = u+iv, the sum and difference are easy:

$$z + w = (x+iy) + (u+iv) = (x+u) + i(y+v) \qquad (6)$$
$$z - w = (x+iy) - (u+iv) = (x-u) + i(y-v) \qquad (7)$$

The product is a little harder, and uses the fact that $i^2 = -1$: $zw = (x+iy)(u+iv) = (xu+i^2yv) + i(xv+yu) = (xu-yv) + i(xv+yu)$, so:

$$zw = (xu-yv) + i(xv+yu) \qquad (8)$$

The trick for computing the quotient z/w is "realizing the denominator": multiplying numerator and denominator by the conjugate u-iv of the denominator, thus transforming the denominator into a pure real, by which we can divide through: $z/w = (x+iy)/(u+iv) = (x+iy)(u-iv)/[(u+iv)(u-iv)] = [(xu+yv) + i(yu-xv)]/(u^2+v^2) = (xu+yv)/(u^2+v^2) + i(yu-xv)/(u^2+v^2)$, so:

$$z/w = (xu+yv)/(u^2+v^2) + i(yu-xv)/(u^2+v^2) \qquad (9)$$

"Really elaborate formulae" usually just involve repeated application of these four rules, (6) - (9). As a particular example, consider the iteration used in computing views of the Mandelbrot set, $z \mapsto z^2+c$. Letting c = u+iv: $z^2+c = (x+iy)(x+iy) + (u+iv) = (x^2-y^2) + 2ixy + (u+iv) = (x^2-y^2+u) + i(2xy+v)$:

$$z^2+c = (x^2-y^2+u) + i(2xy+v) \qquad (10)$$

so the real and imaginary components of the iteration are:

$$x \mapsto x^2-y^2+u \qquad (11)$$
$$y \mapsto 2xy+v \qquad (12)$$

which can be used to compute the iteration.

TUTORIAL: INEQUALITIES

The material in this and the previous tutorial (TUTORIAL: CONJUGATION) follows Ahlfors' Complex Analysis pretty closely, so if you are interested in this material and want to pursue it more deeply, get a copy of that book (see MAIL ORDER BOOKS, page 8 of this issue).

Although there is no order relation between complex numbers, certain relationships of order hold between real numbers associated with them: real part, imaginary part, modulus.

In the previous issue we defined the modulus |z| of a complex number z = a+ib to be the non-negative square root of a^2+b^2; thus:

$$-|z| \le Re(z) \le |z| \qquad (1)$$
$$-|z| \le Im(z) \le |z| \qquad (2)$$

Re(z) = |z| holds if and only if z is real and ≥ 0.

In the previous tutorial we established (the formula there unfortunately had an error):

$$|z+w|^2 = |z|^2 + |w|^2 + 2\cdot\mathrm{Re}(zw^*) \qquad (3)$$

so by (1), $|z+w|^2 \le |z|^2 + |w|^2 + 2\cdot|zw| = (|z| + |w|)^2$, hence

$$|z+w| \le |z| + |w| \qquad (4)$$

This is called the *triangle inequality.*

We can extend (4) to arbitrary sums by applying mathematical induction:

$$|z_1+z_2+...+z_n| \le |z_1| + |z_2| + ... + |z_n| \qquad (5)$$

The modulus of a sum is at most equal to the sum of the modulus of the terms.

When does equality hold in (5)? It holds in (4) if and only if $zw^* \ge 0$[1]. If $w \ne 0$ the inequality can be written $|w|^2(z/w) \ge 0$, which is equivalent to $z/w \ge 0$. Equality holds in the general case (5) if and only if the ratio of any two nonzero terms is positive.

(4) implies

$$|z| \le |(z-w) + w| \le |z-w| + |w|$$

or

$$|z| - |w| \le |z - w|.$$

Similarly $|w| - |z| \le |z - w|$, and these two inequalities can be combined:

$$|z - w| \ge ||z| - |w|| \qquad (6)$$

A special case of (4) is

$$|a + ib| \le |a| + |b| \qquad (7)$$

i.e. the modulus of a complex number is at most the sum of the moduli of its real and imaginary parts.

A more subtle relation (which we will not attempt to prove here) is *Cauchy's inequality:*

$$|z_1w_1 + ... z_nw_n|^2 \le (|z_1|^2 + ... |z_n|^2)(|w_1|^2 + ... |w_n|^2) \qquad (8)$$

Equality holds in (8) if and only if the z_i are proportional to the w_i^*, that is, for some (complex) λ:

$$z_i = \lambda w_i^*, i = 1...n.$$

In fact $\lambda = (z_1w_1 + ... z_nw_n)/(|w_1|^2 + ... + |w_n|^2)$.

SLIDE CORRIGENDA

—*Ken Philip*

You made one error in listing my slides. #375 (which I call the 'pinwheel') is a Mandelbrot, not a Julia. The coordinates are: r from -0.1986162765 to -0.1986162737, i from +1.1003460650 to +1.1003460669. Magnification is 1 billion diameters. Escape radius 100, max dwell 240. Colors produced by changing the original colors with the Mac II DA 'Klutz'.

#347 is essentially the same figure as the slide I sent you recently called -1.71115c [#458], at a slightly lower magnification (perhaps 250x), done with Mandel881. This is the -1.71115 midget on the spike.

#343 I don't have coordinates for offhand, but it's another of the Mandelbrot plots from Mandel881 that I altered the colors of with Klutz.

Also, I thought I had given you the size in pixels (roughly 640x440, except 620x420 for the Mandel881 plots, on the screen — slightly smaller in the slides), and the machine (Mac II with Apple color monitor).

LETTERS

—*from George Zopf*

February 11, 1988

Dear Rollo,

I am sure I am way overdue on my subscription payment, so I enclose another $15.00 for another progression of AMYGDALA. I continue to enjoy it, though I am nowhere near as mandelbred as most of your readers appear to be. I have had a great deal of wonder and aesthetic pleasure from the latest Mandelbrot program to appear on the Fred Fish public domain disks for the Commodore Amiga. This is "IMandelVroom", and is on FF #90. It makes good use of Amiga color graphics.

Now a blast: you solicit $8K to purchase a Macintosh II with color monitor, to continue the Great Work. I have no objection at all to your continuing the great work, nor to your asking for money to do so. Nor can I complain about the quality and capability of the Macintosh II with color-graphic periferals. But if you go that particular route, you are wasting at least two-thirds of the money expended. There is no doubt that both IBM and Apple manufacture good and reliable products; often quite as good as the best. But both companies charge an unconscionable premium for their perceived eminence, and eminence that is in no way reflected in the actual quality and performance of their products. I can understand why the dimmos of industry and commerce might sucker into the belief that "it must be good, because it costs so much", but I am dismayed that otherwise intelligent individuals (and presumably "intelligent" institutions like schools and research institutes) actually go with the herd.

For years now, IBM has clearly demonstrated that it is in the business of making money: the fact that it does so by selling computers is incidental. Apple, over the years, has capitalized on being the "little guy", brash, bright, innovative, friendly, a yuppie-hippy David against a Goliath in a three-piece suit. But both are alike in their gross appeal to snobbery; both maintain prices that are immorally above cost, and certainly out of proportion to any demonstrable functional superiority of their products.

I know I can't really scold you for preferring the Macintosh, any more than I can scold my brother-in-law for clinging to Cadillac. But the cases are almost comparable: the

[1] When we say $z > 0$ for a complex number z we mean that z is *real* and *positive*.

Cadillac *has* real features of superiority.

grumpily,
George

—from Rita Plukss

Dear Rollo,

I am part of a small group of Atari ST users in Melbourne Australia who have become fascinated with Mandelbrots, Julia sets, chaotic attractors and fractals in general over a number of years. Although we generally lack the rigorous mathematical background shown by many of your authors we nevertheless do have fun and appreciate the aesthetic quality of our products. As there are very few commercial Mandelbrot generators available for the Atari ST we have written our own programs including one that links successive screens to create a movie loop effect.

If you have any listings for the Atari ST or could provide contacts with other Atari Mandelbrot enthusiasts I would appreciate hearing from you. Conversely, if you know of other Atari ST users seeking Mandelbrot companionship you may pass my name and address to them.

Yours faithfully
Rita Plukss
13 Stokes Place
Eltham 3095
Victoria
AUSTRALIA

—from Wilfred H. Ward

February 12, 1988

Dear Mr. Silver:

From sublime to ridiculous: Has anyone discussed what happens if one tries THE algorithm backward? To wit:

$$z \mapsto \sqrt{z - c}$$

I figure I should be able to do the square root of a complex number easily by De Moivre's theorem, but that math is long unused.

RS replies:

John Dewey Jones has investigated $z \mapsto z^n + c$ for some non-integral n. The slide sent out with Amy #3 illustrated n = 2.5. In that same issue, I discussed (BIFURCATIONS) some of the problems that arise when you mess with non-integral n.

Yeah, you could compute the square root of $z = re^{i\theta} = r\cos(\theta) + ir\sin(\theta)$ using the formula

$$z^{1/2} = r^{1/2}\cos(\theta/2) + ir\sin(\theta/2)$$

but you also have the other solution to deal with:

$$z^{1/2} = -r^{1/2}\cos(\theta/2) - ir\sin(\theta/2)$$

Do you deal with both? If not, how do you choose one over the other? Insufficient attention paid to these details led to the "fracture lines" that appear in slide #167 and others of that type.

I suspect that $z \mapsto (z - c)^{1/2}$ will prove somewhat dull, since, for one thing, ∞ is not an attractor. I don't really know how it will turn out, though. If it does prove interesting (or not), please let us know!

THE INTERIOR OF $\mathcal{M}$, ONCE AGAIN

George Zopf sent me the following material which was written by one Howard Hull.

Ever wonder what evil lurks in the blackened depths of the inner circle of the Mandelbrot set? Long ago, when Robert French came out with the first widely distributed C language Mandelbrot set generator for the Amiga, I attempted to hack it to show what was going on in the inner reaches with a set of contours just like the outer Mandelbrot. I failed.[1] My reasoning went something like this: The Mandelbrot fractal boundary is considered to be "stable". The points along the boundary are not migrating in the accumulated formulary sum. Points outside the boundary flee to infinity. Points inside the boundary fall to various limit cycles or to certain loci in the inner map, such as x=0, y=0. The destination points of the flight are known as "strange attractors".[2] The outer infinite destination is a strange attractor at infinity. The one at the origin is perhaps easy to accept — the iterative product of real fractions, at least, ought to go to zero as a limit. The other points are elusive rascals to locate if one has no prior heuristic knowledge of likely sites for loci. The original Amiga *Mand.c* program, as improved by R.J. Mical had a small section of inescapable code (you had to reboot to get out of it) called the "Analyzer" with which you could, after executing "SA filename" and "L filename" followed by "A", examine the orbits of the Mandelbrot sum points on the fly. By leaning on the left mouse button and patrolling with the cursor, one could find the strange attractors (I think they're the places where the know of points condenses into a minimum number of clusters of minimal diameter — one in the main body, three in the lobes at the top and bottom, four in the first lobe on the x axis, five for some intermediate lobes, etc.).

Well, anyway, back to the matter at hand. The cause of my failure was rooted in the following reasoning: If the fleeing points are advancing toward what is nominally considered a bye-bye level of absolute radial magnitude 2.0 then we could suppose that points bound to collapse toward the inside may be considered doomed when they fall to absolute magnitude 0.5, right? So I set up a little *if* statement to trap this condition. Much to my surprise and horror, I found that many of the points destined to escape to the *outside* did so by orbiting through radial magnitudes 0.5 and lower. In fact, setting the separator as low as 0.03125 left zillions of points improperly sorted. I was forced to conclude that there was

[1] See Amygdala #4, page 1: **EXPLORING THE INTERIOR OF $\mathcal{M}$.**

[2] Is this right, you dynamical hackers? I think perhaps "**strong** attractors", or maybe simply "attractors" is the right term.

no practical lower value that would sort the inwardly collapsing points. And with that, I went on to other business and left the strange attractors to the math experts.

Recently, there was Yet Another Mandelbrot Program, titled *MandelVroom,* posted to the net by one Kevin Clague at Amdahl. He added a Motorola floating point section that he claimed would give much improved resolution over the previously available Mandelbrots, though I haven't checked that out in detail yet. One thing that Kevin did make a note about in his code is the "Ring Detector" for ponderous points in the inner lobes of the Mandelbrot interior exo-set. This code detects non-migrating orbits for rotating points and escapes to more productive duty, setting the cell count to max on the way out. However, another thing it is capable of doing, so it turns out, is acting as a strange attractor contour generator of sorts. It doesn't do a perfect job of this, (in fact, I am wondering why it works at all) in that some places where there are inflections in the contour[3] it just gets "noisy". Nonetheless, the contours produced may be of interest to some of you. And why not? The change to the source involves commenting out only three lines of assembly code. The results will not hold your attention as well as does the Mandelbrot set, but (Yawn) it's something to know about... In *mand.c* I put semicolons at the beginning of the lines as shown below:

At line 229 in *mand.c* find the ring detector loop and add the lines flagged with semicolons:

```
lloop1
        cmp.l       (a0)+,d4
        bne         skipit
        cmp.l       (a1)+,d5
        bne         next1
;       move.w      _MaxCount,d0
;       ext.l       d0
;       move.l      d0,k(a5)
;       move.l      d0,l(a5)
;       bra         out
```

If you make the patch, remember to select FFP in the Generator item submenu after opening the EDIT menu. For those with no Manx, I'll post a .uue that has this done for you, (along with an optimized color register set) in a following article.

RS: The following comment apparently accompanies some code posted somewhere... on "the net"? I'm supposing it is of interest to you Amiga hackers out there.

This is a very slightly modified copy of the Kevin Clague *MandelVroom* Mandelbrot generator program. It uses his "ring detector" to draw an approximation of the interior strange attractor contours. Even though this article's length is less than 65K (and thus it is not a finkledrill object) the uuencode program used to package it was "uuchecksum", sent to the net by Alan J. Rosenthal. I don't know if that program is the same as the one on the fish disk. At any rate, any uudecode will recreate the binary executable. The executable should be 43,116 bytes.

[If yet unproven concepts are outlawed in the range of discussion... then only the deranged will discuss yet unproven concepts.]

Howard Hull

RS notes the following:

From Deterministic Chaos, by Heinz Georg Schuster, pp. 91, 93-94:

$$\dot{\vec{x}} = \vec{F}(\vec{x}),\ \vec{x} = (x_1, x_2, \ldots x_d) \qquad (5.1)$$

A *strange attractor* has the following properties:

a) It is an attractor, i.e. a bounded region of phase space to which all sufficiently close trajectories from the so-called basin of attraction are attracted asymptotically for long enough times. We not that the basin of attraction can have a very complicated structure (see the pictures in Sect. 5.7). Furthermore the attractor itself should be indecomposable, i.e. the trajectory should visit every point on the attractor in the course of time. A collection of isolated fixed points is no single attractor.

b) The property which makes the attractor strange is the sensitive dependence on the initial conditions, i.e. despite the contraction of volume [in phase space], lengths need not shrink in all directions, and *points which are arbitrarily close initially become macroscopically separated at the attractor after sufficiently long times...*

c) To describe a physical system, the attractor has to be *structurally stable* and *generic*. In other words, a small change in the parameters of $\vec{F}$ in (5.1) changes the structure of the attractor continuously, and the set of parameters for which (5.1) generates a strange attractor should not be of measure zero — otherwise it would be nongeneric and not physically meaningful.

[3] See contour map, Figure 33 on page 60, *The Beauty of Fractals.*

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